QUANTUM MECHANICS

Three Hours
Do Five Problems
Open notes and one text allowed

1. A particle of mass m moves in the one-dimensional potential

$$V(x) = \alpha|x| \quad \text{for } |x| \leq L$$
$$= \infty \quad \text{for } |x| > L$$

Describe the approximate structure of the low-lying energy levels, E_n, in the two limiting situations where the quantity $\frac{\hbar^2}{m\alpha L^3}$ is $\gg 1$ and $\ll 1$, respectively.

2. Suppose a potential falls off at large distances as r^{-n}. Use the first Born Approximation for the scattering amplitude to find the required condition on n in order for the total elastic scattering to be finite.

3. Consider two spin-½ systems bound together in a manner approximating a rigid diatomic molecule. Assume there is a relatively weak spin-orbit coupling of the form $a\,\vec{L}\cdot\vec{S}$, where $\vec{L}$ = orbital angular momentum, $\vec{S}$ = total spin angular momentum of the system, and $a \ll \frac{1}{I}$ where I is the moment of inertia of the molecule.

   (a) Denoting the states by the usual spectroscopic notation, describe the gross structure and fine structure of the levels of the system.

(b) Assume that in addition there is a weak spin-spin interaction of the form $b\,\vec{\sigma}_1 \cdot \vec{\sigma}_2$, with $b \ll a$. What is the primary extra feature introduced into the level structure by this term?

(c) On a clearly labeled energy level diagram indicate the lowest energy electric dipole radiative transitions; giving the fine structure of the lowest frequency line.

4. (a) Suppose the proton of a tritium atom (H^3) undergoes a decay in which the proton disappears and its decay products escape rapidly without perturbing the rest of the system. Using an appropriate approximation describe as quantitatively as possible the observable final distribution of the atomic electron.

(b) Consider the energy loss due to ionization by a fast charged particle interacting with atomic electrons in a medium. Using a reasonable approximation, make a rough estimate of the maximum collision impact parameter which contributes effectively to the energy loss.

5. A conjectured phenomenon of considerable current interest is "neutron-antineutron oscillation". Suppose the Hamiltonian has a matrix element ϵ connecting the state of a neutron at rest with a given spin direction to the state of an antineutron at rest with the same spin direction. a) If the system consists of a neutron at time $t = o$, what is the probability of finding an antineutron at arbitrary time t? b) The neutron and antineutron have magnetic moments of opposite sign. Suppose a magnetic field is present and at $t = o$ a neutron is in its lowest energy state.

How is the result in part (a) modified? Find an expression for the maximum probability of finding an antineutron.

6. Suppose that in the partial wave expansion only terms up to ℓ_{max} contribute to the elastic cross-section, σ_e, and total cross-section, σ_t, for a certain scattering process; all higher terms are negligible. Find an expression for the minimum value of σ_e subject to the constraint that σ_t is fixed. Use this to find an inequality relating ℓ_{max} to σ_e, σ_t.